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B. TECH.
(SEM I) THEORY EXAMINATION 2022-23
MATHEMATICS-I

Time: 3 Hours ***Total Marks: 100***

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 10 = 20**

a. Find the value of $'b'$ for which the rank of the matrix $A=\begin{bmatrix}1 & 5 & 4\\ 0 & 3 & 2\\ b & 13 & 10\end{bmatrix}$ is 2.

b. The Eigen values of A are $2,3,1$ then find the Eigen values of A^2+A.

c. State Roll's Theorem.

d. If $y=\log x^3$ then find y_n.

e. What is the functional relation between $u=\frac{x}{y}$ and $v=\frac{x+y}{x-y}$.

f. Compute an approximate value of $(1.04)^{3.01}$

g. Evaluate $\int_0^1\int_0^x xy\,dy\,dx$

h. Evaluate the area enclosed between the parabola $y=x^{\mathbf{2}}$ and the straight line $y=x$

i. Prove that if $\vec{u}$ and $\vec{v}$ are irrotational then $\vec{u}\times\vec{v}$ is solenoidal.

j. Find $\nabla(\log r)$ if $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ is position vector.

SECTION B

2. Attempt any *three* of the following: **10x3=30**

a. Reduce the following matrices to its normal (or canonical) form and find the rank:

$$\begin{bmatrix}1 & 2 & -1 & 4\\ 2 & 3 & 3 & 4\\ 1 & 2 & 3 & 4\\ -1 & -2 & 6 & -7\end{bmatrix}$$

b. If $x=\sin(\sqrt{y})$ then find $(y_n)_0$.

c. A rectangular box, open at the top, is to have a volume of $32\,c.c$. Find the dimensions of the box requiring least material for its construction.

d. Evaluate: $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}}\frac{1}{\sqrt{1-x^2-y^2-z^2}}dzdydx$

e. Find the constants a and b such that the curl of a vector $\vec{A}=(2xy+3yz)\hat{i}+(x^2+axz-4z^2)\hat{j}+(3xy+2byz)\hat{k}$ is zero.

SECTION C

3. Attempt any *one* part of the following: 10x1=10

a. Find the inverse of the following matrices:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

b. Find the Eigen values and Eigen vectors of the matrix

$$A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

4. Attempt any *one* part of the following: 10x1=10

a. If $u=f(y-z,z-x,x-y)$, Prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

b. Trace the following curves:
$y^2(2a-x)=x^3$

5. Attempt any *one* part of the following: 10x1=10

a. Expand $\sin^{-1}x$ up to four terms in powers of x.

b. If u,v,w are the roots of the equations $(x-a)^3+(y-b)^3+(z-c)^3=0$, then find $\frac{\partial(u,v,w)}{\partial(a,b,c)}$.

6. Attempt any *one* part of the following: 10x1=10

a. Change the order of integration in $\int_0^a\int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ hence evaluate the same

b. Evaluate by changing the variables, $\iint_R (x+y)^2\,dx\,dy$ where R is the region bounded by parallelogram $x+y=0, x+y=2, 3x-2y=0, 3x-2y=3$

7. Attempt any *one* part of the following: 10x1=10

a. Evaluate $\iint_S (a^2x^2+b^2y^2+c^2z^2)^{1/2}\,dS$ where S is the surface of the ellipsoid $ax^2+by^2+cz^2=1$.

b. Find the directional derivative of $\phi=5x^2y-5y^2z+\frac{5}{2}z^2x$ at the point $P(1,1,1)$ in the direction of the line $\frac{x-1}{2}=\frac{y-3}{-2}=\frac{z}{1}$.